

Experiments and Numerical Simulations on Stress-State-Dependence of Ductile Damage Criteria

Michael Brüning, Steffen Gerke and Daniel Brenner

Abstract The paper deals with a series of new experiments and corresponding numerical simulations to be able to study the effect of stress state on damage behavior of ductile metals. In this context, a thermodynamically consistent anisotropic continuum damage model is presented. It takes into account the effect of stress state on damage and failure conditions as well as on evolution equations of damage strains. Different branches of the respective criteria are considered corresponding to various damage and failure mechanisms depending on stress intensity, stress triaxiality and the Lode parameter. Since it is not possible to propose and to validate stress-state-dependent criteria only based on tests with uniaxially loaded specimens for a wide range of stress states, new experiments with two-dimensionally loaded specimens have been developed. Corresponding numerical simulations of these experiments show that they cover a wide range of stress triaxialities and Lode parameters in the tension, shear and compression domains. The new series of experiments allow validation of stress-state-dependent functions for the damage criteria and are used to identify parameters of the continuum model.

Keywords Ductile damage · Stress state dependence · Experiments · Numerical simulations

1 Introduction

Modeling of inelastic behavior and fracture of materials employed in various engineering applications is an important topic in solid mechanics, for example, in numerical analyses of complex structural components, in assessment of structural reliability,

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or in optimization of structural design. Based on many experiments it is well known that finite or large inelastic deformations caused by loading of ductile metals and metal alloys are accompanied by damage and local failure mechanisms acting on different scales which lead to macro-failure of structural elements. In particular, under tension dominated loading conditions (high positive stress triaxialities) damage in ductile metals is mainly caused by nucleation, growth and coalescence of voids whereas under shear and compression dominated conditions (small positive or negative stress triaxialities) evolution of micro-shear-cracks is the predominant damage mechanism. Therefore, to be able to develop a realistic, accurate and efficient phenomenological model it is important to analyze and to understand the complex stress-state-dependent processes of damage and fracture as well as its respective mechanisms acting on different scales. In this context, in the last years various damage models have been published based on experimental observations as well as on multi-scale approaches (Brünig 2003a; Gurson 1977; Lemaitre 1996; Voyiadjis and Kattan 1999). In this context, Brünig et al. (2008, 2011b), Brünig and Gerke (2011) have proposed a generalized and thermodynamically consistent, phenomenological continuum damage model which has been implemented as user-defined material subroutines in commercial finite element programs allowing analyses of static and dynamic problems in differently loaded metal specimens. To be able to detect stress triaxiality dependence of the constitutive equations tension tests with carefully designed specimens have been developed. For example, differently pre-notched specimens and corresponding numerical simulations have been used by Bai and Wierzbicki (2008), Bao and Wierzbicki (2004), Becker et al. (1988), Bonora et al. (2005), Brünig et al. (2008, 2011b), Dunand and Mohr (2011), Gao et al. (2010). However, these experiments with unnotched and differently notched flat specimens showed stress triaxialities only in a small region of positive values. Larger triaxialities appear in tension tests with cylindrical (axi-symmetric) specimens but they cannot be manufactured when the behavior of thin sheets is investigated. Therefore, it is necessary to develop new series of experiments with flat specimens where a larger range of stress triaxialities will occur.

Thus, specimens with new geometries have been designed to be able to analyze stress states with small hydrostatic parts. Tension tests with these specimens have been performed (Bao and Wierzbicki 2004; Gao et al. 2010) leading to shear mechanisms in their centers. Similar specimens have been developed and tested (Brünig et al. 2008; Driemeier et al. 2010). Furthermore, to be able to take into account other regions of stress triaxialities butterfly specimens have been manufactured (Bai and Wierzbicki 2008; Dunand and Mohr 2011; Mohr and Henn 2007) which can be tested in different directions using special experimental equipment. Alternatively, in the present paper series of new tests with biaxially loaded new flat specimens taken from thin sheets will be developed leading to experimental results on inelastic behavior, damage and fracture of ductile metals for a wide range of stress triaxialities not obtained by the experiments discussed above.

Further information on damage and failure mechanisms can be obtained by performing numerical simulations on the micro-level (Brocks et al. 1995; Brünig et al. 2011a, 2013, 2014; Chew et al. 2006; Kuna and Sun 1996; Needleman and Kushner

1990; Zhang et al. 2001) considering individual behavior of growth and coalescence of voids and micro-shear-cracks as well as their accumulation to macro-cracks. The numerical results elucidated which parameters had remarkable effect on macroscopic stress-strain relations and on evolution equations for the damage variables and which ones only had marginal influence. With these numerical studies taking into account a large range of stress states it was possible to detect different damage mechanisms which have not been exposed by experiments. The equations for damage criteria and damage evolution equations proposed by analyzing in detail the numerical results of the micro-mechanical studies considering differently loaded micro-defect containing representative volume elements showed remarkable dependence on stress triaxiality and—especially in regions with small or negative triaxialities—additional dependence on the Lode parameter or third deviatoric stress invariant. However, the proposed functions are only based on numerical simulations on the micro-level with void-containing unit-cells and, therefore, further experiments with carefully designed specimens are necessary for their validation.

In the present paper fundamental governing equations of the phenomenological continuum damage model proposed by Brünig (2003a) are briefly discussed. Experiments on inelastic and damage behavior of an aluminum alloy are performed. Uniaxial tension tests are used to identify basic elastic-plastic material parameters. Furthermore, newly developed biaxial experiments with 2D-specimens up to final fracture will be presented. These combined shear-tension and shear-compression tests cover a wide range of stress states. Corresponding numerical simulations of these biaxial experiments will reveal various stress measures in critical regions. Experimental and numerical data are used to propose and to validate damage criteria as well as to identify corresponding constitutive parameters.

2 Continuum Damage Model

Irreversible material behavior and anisotropic damage of ductile metals are predicted by Brünig's continuum model (Brünig 2003a). It is based on series of experimental results and observations and, additionally, takes into account information of microscopic mechanisms due to individual micro-defects and their interactions. The phenomenological approach is based on the introduction of damaged and corresponding fictitious undamaged configurations and has been implemented into finite element programs. An extended version of this model takes into account a stress-state-dependent damage criterion based on experimental results of different tension and shear tests with smooth and pre-notched specimens as well as on data from corresponding numerical simulations (Brünig et al. 2008, 2011b). Furthermore, numerical analyses using unit cell models have been performed (Brünig et al. 2011a, 2013, 2014). Based on their numerical results covering a wide range of stress states they proposed damage equations as functions of the stress intensity, the stress triaxiality and the Lode parameter and estimated micro-mechanically based material parameters.

The kinematic approach of the continuum model is based on the introduction of initial, current and elastically unloaded configurations each defined as damaged and fictitious undamaged configurations, respectively. This leads to the additive decomposition of the strain rate tensor into an elastic ($\dot{\mathbf{H}}^{\text{el}}$), a plastic ($\dot{\mathbf{H}}^{\text{pl}}$) and a damage part ($\dot{\mathbf{H}}^{\text{da}}$) (see Brünig 2003a for further details).

The undamaged configurations are considered to describe the constitutive behavior of the undamaged matrix material. This leads to the effective Kirchhoff stress tensor $\bar{\mathbf{T}}$. In addition, plastic behavior of ductile metals is governed by the yield condition

$$f^{\text{pl}}(\bar{I}_1, \bar{J}_2, c) = \sqrt{\bar{J}_2} - c \left(1 - \frac{a}{c} \bar{I}_1\right) = 0, \quad (1)$$

where $\bar{I}_1 = \text{tr} \bar{\mathbf{T}}$ and $\bar{J}_2 = \frac{1}{2} \text{dev} \bar{\mathbf{T}} \cdot \text{dev} \bar{\mathbf{T}}$ are invariants of the effective stress tensor $\bar{\mathbf{T}}$, c denotes the yield stress of the matrix material and a represents the hydrostatic stress coefficient where a/c is a constant material parameter.

Since only isochoric plastic deformations have been observed in ductile metals the plastic potential function

$$g^{\text{pl}}(\bar{\mathbf{T}}) = \sqrt{\bar{J}_2} \quad (2)$$

depends only on the second invariant of the effective stress deviator which leads to the non-associated isochoric effective plastic strain rate

$$\dot{\mathbf{H}}^{\text{pl}} = \dot{\lambda} \frac{\partial g^{\text{pl}}}{\partial \bar{\mathbf{T}}} = \dot{\lambda} \frac{1}{2\sqrt{\bar{J}_2}} \text{dev} \bar{\mathbf{T}} = \dot{\gamma} \bar{\mathbf{N}}. \quad (3)$$

In Eq. (3) $\dot{\lambda}$ is a non-negative scalar-valued factor,

$$\bar{\mathbf{N}} = \frac{1}{\sqrt{2\bar{J}_2}} \text{dev} \bar{\mathbf{T}}$$

represents the effective normalized deviatoric stress tensor and

$$\dot{\gamma} = \bar{\mathbf{N}} \cdot \dot{\mathbf{H}}^{\text{pl}} = \frac{1}{\sqrt{2}} \dot{\lambda}$$

characterizes the equivalent plastic strain rate measure used in the present continuum model.

Moreover, the damaged configurations are considered to describe the constitutive behavior of the damaged material (material sample including micro-defects). It is well known that damage remarkably affects the elastic behavior and leads to deterioration of elastic material properties. The corresponding elastic law of the damaged material leads to the Kirchhoff stress tensor $\mathbf{T}$ (see Brünig 2003a for further details). In addition, constitutive equations for damage evolution are required and the determination of onset and continuation of damage is based on the concept of damage

surface formulated in stress space at the macroscopic damaged continuum level. The damage condition

$$f^{\text{da}} = \alpha I_1 + \beta \sqrt{J_2} - \sigma = 0 \quad (4)$$

is expressed in terms of the stress invariants of the Kirchhoff stress tensor, $I_1 = \text{tr} \mathbf{T}$ and $J_2 = \frac{1}{2} \text{dev} \mathbf{T} \cdot \text{dev} \mathbf{T}$, and the damage threshold σ . In Eq. (4) the variables α and β denote damage mode parameters depending on the stress intensity $\sigma_{\text{eq}} = \sqrt{3J_2}$, the stress triaxiality

$$\eta = \frac{\sigma_m}{\sigma_{\text{eq}}} = \frac{I_1}{3\sqrt{3}J_2} \quad (5)$$

defined as the ratio of the mean stress σ_m and the von Mises equivalent stress σ_{eq} as well as on the Lode parameter

$$\omega = \frac{2T_2 - T_1 - T_3}{T_1 - T_3} \quad \text{with } T_1 \geq T_2 \geq T_3, \quad (6)$$

expressed in terms of the principal Kirchhoff stress components T_1 , T_2 and T_3 (Brüning et al. 2013).

Furthermore, increase in macroscopic irreversible strains caused by the simultaneous nucleation, growth and coalescence of micro-defects is modeled by a stress-state-dependent damage rule. In this context, the damage potential function

$$g^{\text{da}}(\tilde{\mathbf{T}}) = g^{\text{da}}(I_1, J_2, J_3), \quad (7)$$

is introduced where $\tilde{\mathbf{T}}$ represents the stress tensor formulated in the damaged configuration which is work-conjugate to the damage strain rate tensor $\dot{\mathbf{H}}^{\text{da}}$ (see Brüning 2003a for further details) and I_1 , J_2 and J_3 are corresponding invariants which coincide with those of the Kirchhoff stress tensor. This leads to the damage strain rate tensor

$$\dot{\mathbf{H}}^{\text{da}} = \dot{\mu} \frac{\partial g^{\text{da}}}{\partial \tilde{\mathbf{T}}} = \dot{\mu} \left(\frac{\partial g^{\text{da}}}{\partial I_1} \mathbf{1} + \frac{\partial g^{\text{da}}}{\partial J_2} \text{dev} \tilde{\mathbf{T}} + \frac{\partial g^{\text{da}}}{\partial J_3} \text{dev} \tilde{\mathbf{S}} \right) \quad (8)$$

where $\dot{\mu}$ is a non-negative scalar-valued factor and

$$\tilde{\mathbf{S}} = \text{dev} \tilde{\mathbf{T}} \text{dev} \tilde{\mathbf{T}} - \frac{2}{3} J_2 \mathbf{1} \quad (9)$$

represents the second order deviatoric stress tensor. Alternatively, the damage strain rate tensor (8) can be written in the form

$$\dot{\mathbf{H}}^{\text{da}} = \dot{\mu} \left(\bar{\alpha} \frac{1}{\sqrt{3}} \mathbf{1} + \bar{\beta} \mathbf{N} + \bar{\delta} \mathbf{M} \right) \quad (10)$$

where the normalized tensors

$$\mathbf{N} = \frac{1}{2\sqrt{J_2}} \text{dev}\tilde{\mathbf{T}}$$

and

$$\mathbf{M} = \frac{1}{\|\text{dev}\tilde{\mathbf{S}}\|} \text{dev}\tilde{\mathbf{S}}$$

have been used. In Eq. (10) the parameters $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\delta}$ are kinematic variables describing the respective portion of volumetric and isochoric damage-based deformations. The damage rule (10) takes into account isotropic and anisotropic parts corresponding to isotropic growth of voids and anisotropic evolution of micro-shear-cracks, respectively. The inclusion of the isochoric terms in Eq. (10) rests on the notion that the volume of micro-defects undergoing shear dominated loadings may not significantly increase but deformation of micro-defects and their reorientation also constitute an effective increase in damage and will, therefore, contribute to softening of material behavior. Thus, the damage rule (10) is able to model the stress-state-dependent mechanisms discussed above.

Stress-state-dependence of the parameters in the damage condition (4) and in the damage rule (8) has been investigated in detail performing numerical simulations on the micro-level (Brünig et al. 2013). Based on various unit-cell calculations taking into account a wide range of stress triaxiality coefficients η and Lode parameters ω the damage mode parameter α in the damage criterion (4) has been proposed to be

$$\alpha(\eta) = \begin{cases} 0 & \text{for } -\frac{1}{3} \leq \eta \leq 0 \\ \frac{1}{3} & \text{for } \eta > 0 \end{cases} \quad (11)$$

whereas β is taken to be the non-negative function

$$\beta(\eta, \omega) = \beta_0(\eta, \omega = 0) + \beta_\omega(\omega) \geq 0, \quad (12)$$

see Fig. 1, with

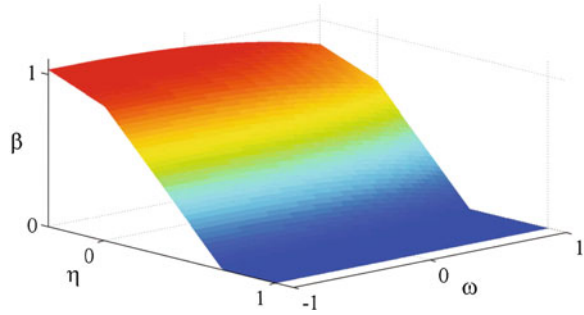
$$\beta_0(\eta) = \begin{cases} -0.45 \eta + 0.85 & \text{for } -\frac{1}{3} \leq \eta \leq 0 \\ -1.28 \eta + 0.85 & \text{for } \eta > 0 \end{cases} \quad (13)$$

and

$$\beta_\omega(\omega) = -0.017 \omega^3 - 0.065 \omega^2 - 0.078 \omega. \quad (14)$$

Figure 1 clearly shows that in the negative stress triaxiality regime $-1/3 \leq \eta \leq 0$ the damage mode parameter β is large signifying remarkable influence of the second deviatoric stress invariant J_2 on onset of damage whereas the influence of the negative hydrostatic stress is small and does not remarkably affect the damage behavior. This corresponds to dominant shear mechanisms and marginal volume changes caused by

Fig. 1 Parameter β versus stress triaxiality η and Lode parameter ω (Brünig et al. 2013)



damage. On the other hand, for high stress triaxialities the damage mode parameter is taken to be $\beta = 0$ and, thus, only the first stress invariant I_1 is taken to determine the onset of damage caused by predominant, nearly isotropic void growth mechanisms. However, this stress-state-dependent damage criterion is only based on numerical unit-cell calculations and, therefore, it will be validated in the present paper by various experiments and corresponding numerical simulations on the macro-level covering a wide range of stress triaxialities and Lode parameters.

3 Identification of Material Parameters

Elastic-plastic material parameters are identified using experimental results from uniaxial tension tests with unnotched specimens. Equivalent stress–equivalent plastic strain curves are easily obtained from load-displacement curves as long as the uniaxial stress field remains homogeneous between the clip gauges fixed on the specimens during the tests. It is worthy to note that equivalent stress–equivalent strain curves especially at large inelastic deformations have to be modeled accurately because material response for multi-axial loading conditions as well as localization phenomena are very sensitive to the identified parameters, especially to the current plastic hardening modulus.

For the aluminum alloy of the series 2017 investigated in the present paper, fitting of numerical curves and experimental data leads to Young's modulus $E = 65,000$ MPa and Poisson's ratio is taken to be $\nu = 0.3$. For the plastic material behavior, the power law function for the equivalent stress–equivalent plastic strain function appearing in the yield criterion (1)

$$c = c_0 \left(\frac{H \gamma}{n c_0} + 1 \right)^n \quad (15)$$

is used to model the work-hardening behavior. Good agreement of experimental data and numerical results is achieved for the initial yield strength $c_0 = 175$ MPa, the hardening modulus $H = 2,100$ MPa and the hardening exponent $n = 0.22$.

In addition, experimental results of notched uniaxial specimens are used to determine the hydrostatic stress dependence of the yield condition (1). Performing elastic-plastic numerical calculations leads to the hydrostatic stress coefficient $a/c = 0.000055$.

Furthermore, onset of damage is determined by comparison of experimental results of tension tests with corresponding elastic-plastic numerical analyses (Brünig et al. 2011b). For the aluminum alloy under investigation the damage threshold appearing in the damage criterion (4) is identified to be $\sigma = 300$ MPa.

4 Experiments with Biaxially Loaded Specimens

The main purpose of the experimental program is to develop and to propose a set of new tests revealing the effect of stress state on damage and failure in ductile metals. The experiments are performed using the biaxial test machine (Type LFM-BIAX 20 kN from Walter and Bai, Switzerland) shown in Fig. 2. It is composed of four electro-mechanically, individually driven cylinders with load maxima and minima of ± 20 kN (tension and compression loading is possible). The specimens are fixed in the four heads of the cylinders where clamped or hinged boundary conditions are possible. The geometry of the newly designed flat specimens and the loading conditions are shown in Fig. 3. The geometry is similar to that one of specimens recently

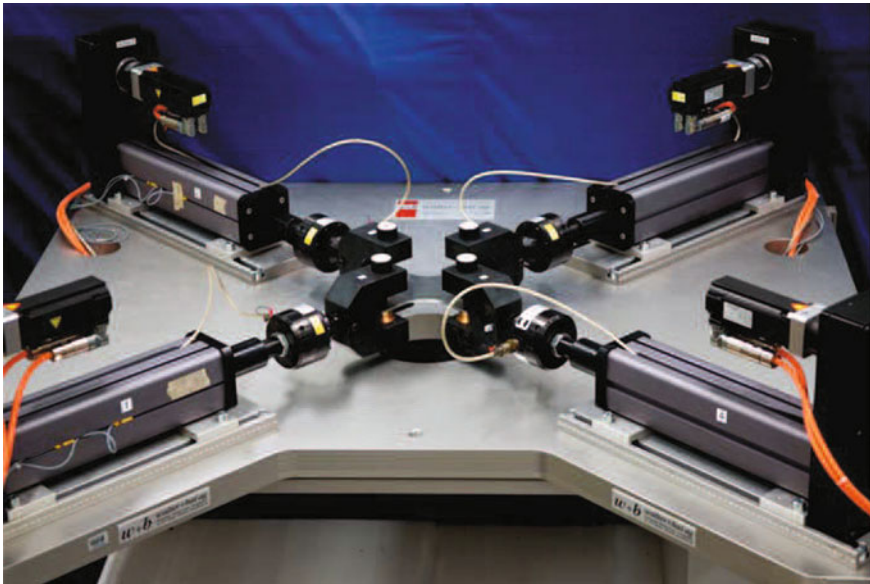
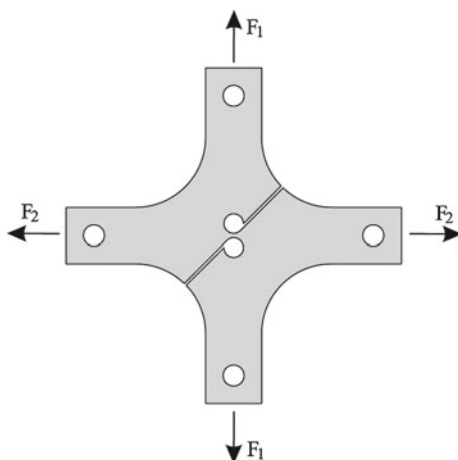


Fig. 2 Biaxial test machine

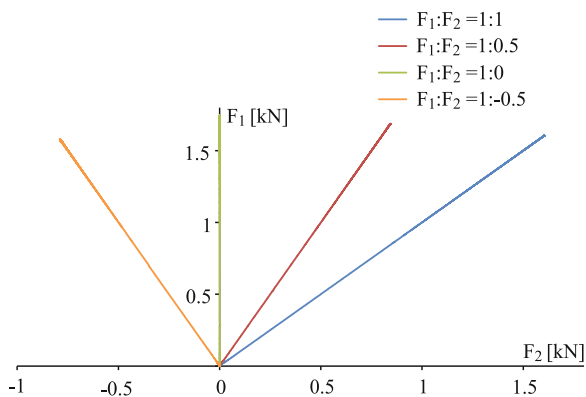
Fig. 3 Specimen and loading conditions



tested firstly in tension and subsequently in shear tests or vice versa (Driemeier et al. 2010), but here the specimens are biaxially strained. The load F_1 will lead to shear mechanisms in the center of the specimen whereas the simultaneous loading with F_2 leads to superimposed tension or compression modes leading to shear-tension or shear-compression deformation and failure modes. Therefore, this extension of experimental work covers the full range of stress states corresponding to the damage and failure mechanisms discussed above with focus on high positive as well as low positive, nearly zero and negative stress triaxialities where the Lode parameter also plays an important role.

In particular, the load ratios $F_1 : F_2$ remain constant during the entire experiments up to final fracture, see Fig. 4. This leads to load-displacement curves shown in Fig. 5 for four different load ratios. Figure 5 clearly shows that the amount of the vertical load F_1 (leading to shear modes) is only marginally affected by the different superimposed horizontal loads F_2 . However, different load ratios $F_1 : F_2$ have

Fig. 4 Vertical load F_1 versus horizontal load F_2



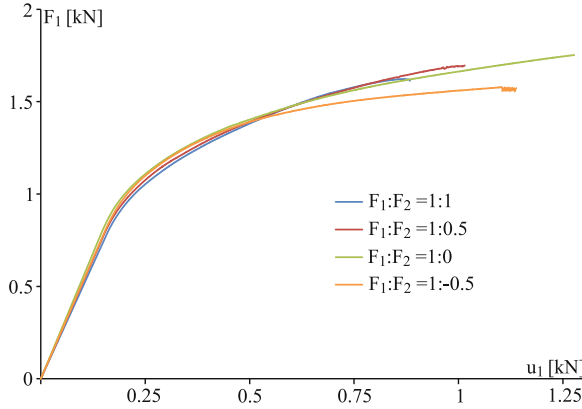


Fig. 5 Load-displacement curves

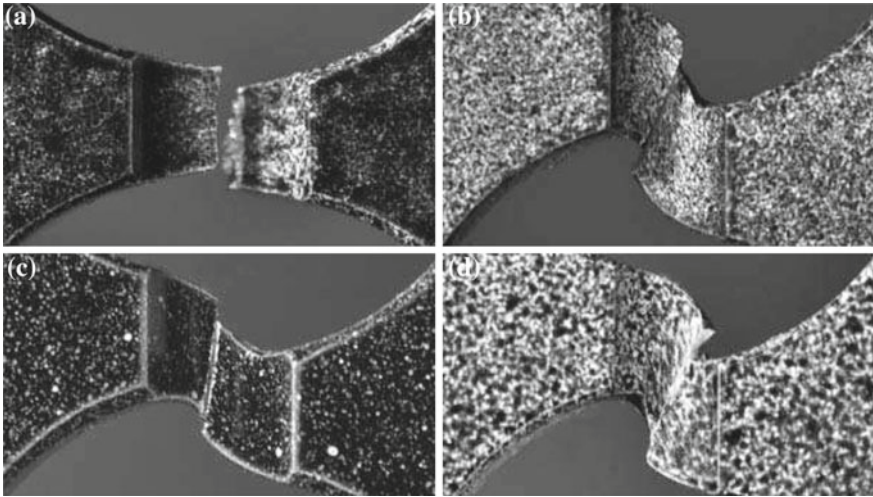


Fig. 6 Fracture modes of differently loaded specimens: **a** $F_1 : F_2 = 0 : 1$, **b** $F_1 : F_2 = 1 : 0$, **c** $F_1 : F_2 = 1 : 0.5$, **d** $F_1 : F_2 = 1 : -0.5$

remarkable influence on the damage and final fracture modes, see Fig. 6. For example, for the tension test without shear loading, $F_1 : F_2 = 0 : 1$, a nearly vertical fracture line is obtained. Under this loading condition, damage is mainly caused by growth and coalescence of voids with small influence of micro-shear-cracks leading to the macroscopic tensile fracture mode with small cup-cone fracture effect (Fig. 6a). On the other hand, for pure shear loading, $F_1 : F_2 = 1 : 0$, shear fracture is observed where the fracture line has an angle of about 25° with respect to the vertical line. Under this loading condition, damage is mainly caused by formation of micro-shear-cracks leading to the macro-shear-crack shown in Fig. 6b. For

shear-tension loading, $F_1 : F_2 = 1 : 0.5$, shear-tension fracture is obtained and the fracture line has an angle of only about 10° with respect to the vertical line. Under this loading condition, damage is caused by the simultaneous growth of voids and formation of micro-shear-cracks leading to the macroscopic fracture mode shown in Fig. 6c which is between tensile fracture (Fig. 6a) and shear fracture (Fig. 6b). And for shear-compression loading, $F_1 : F_2 = 1 : -0.5$, again shear fracture is observed and the fracture line again has an angle of about 25° with respect to the vertical line. Under this loading condition damage seems to be caused only by formation of micro-shear-cracks leading to the macro-shear-crack shown in Fig. 6d. It is worthy to note that the damage and failure mechanisms for $F_1 : F_2 = 1 : 0$ (Fig. 6b) and $F_1 : F_2 = 1 : -0.5$ (Fig. 6d) seem to be very similar and damage modes characteristic for shear loading will not be remarkably affected by superimposed small compression loads.

5 Numerical Simulations of the 2D Experiments

In the experiments with biaxially loaded specimens discussed above the stress and strain fields are not homogeneous and only quantities in an average sense can be evaluated from these tests. Therefore, corresponding numerical simulations have been performed to be able to get detailed information on amounts and distributions of different stress and strain measures as well as further parameters of interest especially in critical regions. The numerical calculations have been carried out using the finite element program ANSYS enhanced by a user-defined material subroutine. This subroutine takes into account numerical integration of the constitutive equations by an extended version of the inelastic predictor–elastic corrector technique and corresponding consistent tangent moduli (Brünig 2003b).

The finite element mesh of the discretized biaxially loaded specimen is shown in Fig. 7. The three-dimensional finite element mesh is based on 42,248 eight-node-elements of the type Solid185. Remarkable refinement of the finite element mesh can be seen in the central part of the specimen where high gradients of the stress and strain related quantities are expected.

Figure 8 shows comparison of experimental and numerically predicted load-displacement curves for the load ratio $F_1 : F_2 = 1 : 1$. The numerical results are based only on an elastic-plastic analysis to be able to identify the onset of damage (Brünig et al. 2011b). In particular, good agreement of experimental and corresponding numerical curve is shown in Fig. 8 for the first part of the biaxial test and only deviation in the last part is observed. This deviation is caused by the occurrence of damage in the experiment which was not taken into account by the elastic-plastic analysis. Thus, this loading stage characterizes onset of damage in the specimen during this test and at this point the stress and strain states in the critical specimen's center will be analyzed in detail. The numerically predicted data for this calculation and also for other loading ratios $F_1 : F_2$ will give information on the effect of stress state on the damage criterion.

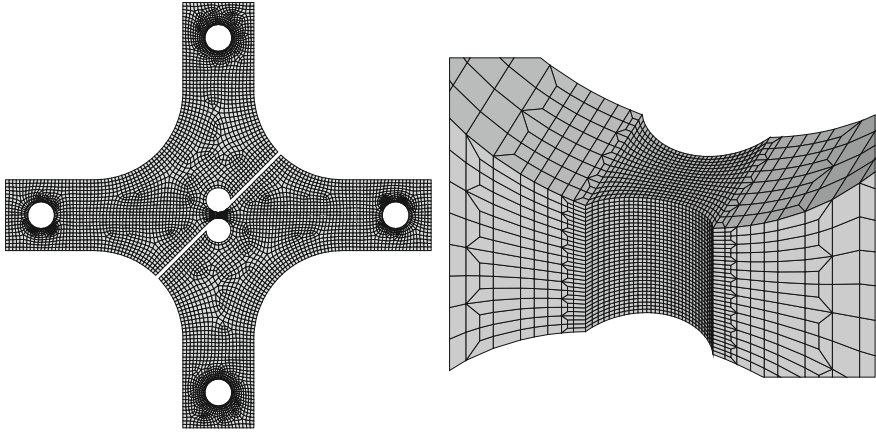


Fig. 7 Finite element mesh of the 2D specimen

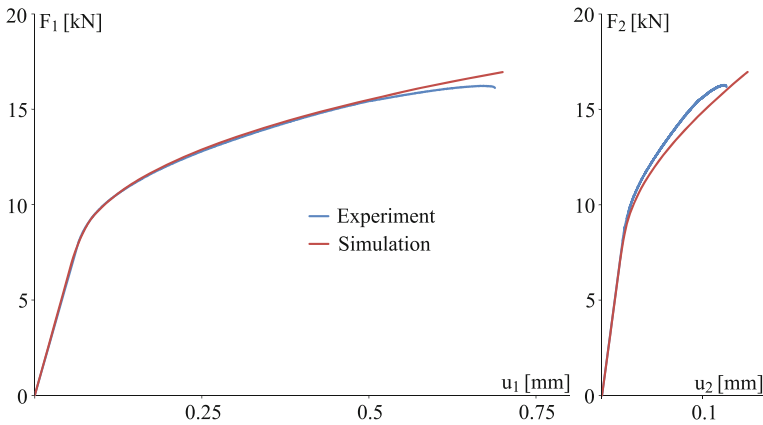


Fig. 8 Load-displacement curves for $F_1 : F_2 = 1 : 1$

For the load ratio $F_1 : F_2 = 1 : 1$, Fig. 9 shows the distribution of the first and second deviatoric stress invariants I_1 and $\sqrt{J_2}$ in the center of the biaxially loaded specimen. In particular, maximum of the first stress invariant I_1 is seen on the boundaries of the notched part near the center of the specimen. In this part, also very high values of the second deviatoric stress invariant $\sqrt{J_2}$ are numerically predicted. Thus, damage will start here caused by growth of voids and micro-shear-cracks. Furthermore, maximum of $\sqrt{J_2}$ can be seen in the center of the specimen and in this region the first stress invariant I_1 is moderate. This means that after onset of damage on the boundaries further damage will also occur here mainly caused by the formation of micro-shear-cracks with small effect of isotropic growth of micro-voids.

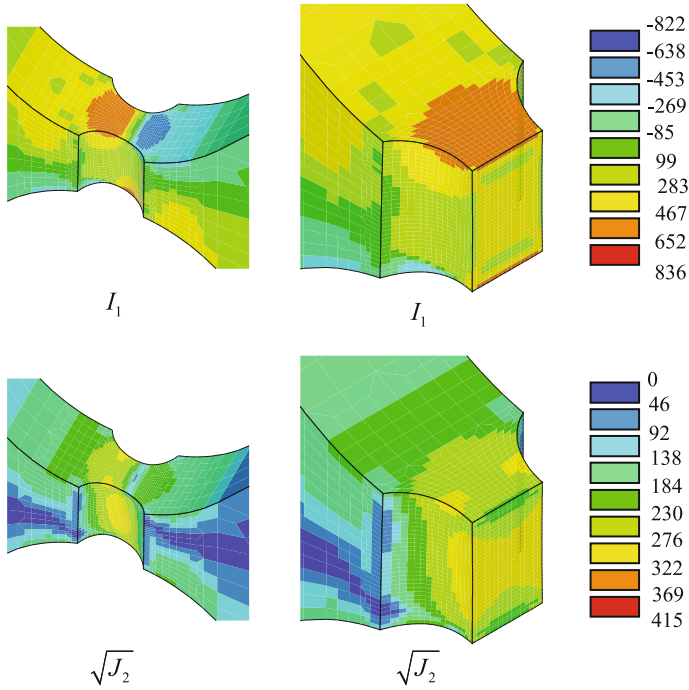


Fig. 9 Distribution of the stress invariants I_1 and $\sqrt{J_2}$ for $F_1 : F_2 = 1 : 1$

The stress state in critical regions of the specimen is remarkably influenced by loading conditions. In this context, numerically predicted distributions of the stress triaxiality η at the onset of damage are shown in Fig. 10 for different load ratios. In particular, for uniaxial tension loading $F_1 : F_2 = 0 : 1$ remarkable high stress triaxialities up to $\eta = 0.84$ are numerically predicted in the center of the specimen. These high values are caused by the notches in horizontal and thickness direction leading to high hydrostatic stress during elongation of the specimen. This will lead to damage and failure mainly due to growth of voids. In addition, when the specimen is only loaded by $F_1 : F_2 = 1 : 0$ (shear loading condition) the stress triaxiality η is numerically predicted to be nearly zero in the whole vertical section shown in Fig. 10. Thus, damage will start in the specimen's center and will be caused by formation and growth of micro-shear-cracks. Furthermore, combined loading in vertical and horizontal direction, F_1 and F_2 , will lead to combination of these basic damage modes. For example, for the load ratio $F_1 : F_2 = 1 : 1$ the stress triaxiality is again nearly constant in the vertical section with $\eta = 0.25$, and very similar distribution is numerically predicted for $F_1 : F_2 = 1 : 0.5$ with $\eta = 0.14$. On the other hand, the load ratio $F_1 : F_2 = 1 : -0.5$ represents combined shear-compression loading with the stress triaxiality $\eta = -0.14$ which is nearly constant in the vertical section shown

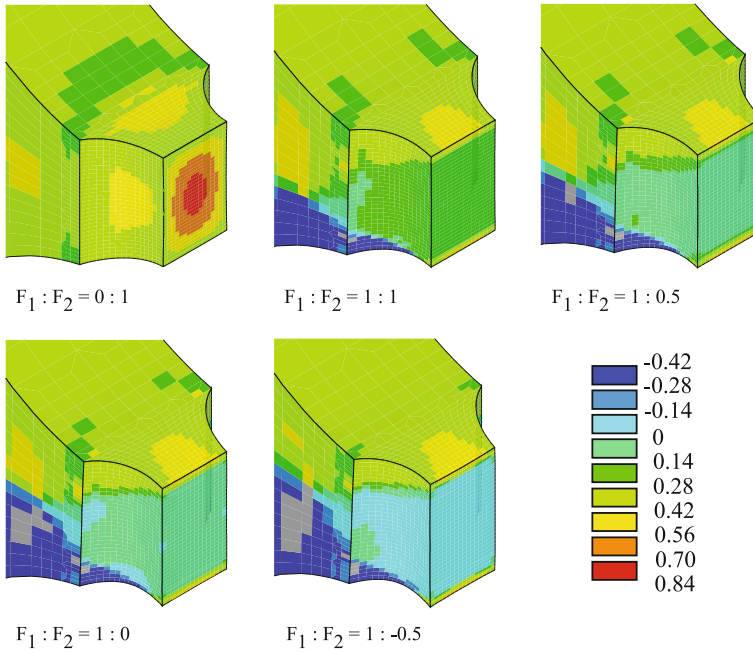


Fig. 10 Distribution of the stress triaxiality η for different load ratios

in Fig. 10. For this negative stress triaxiality damage and failure will be caused by formation and growth of micro-shear-cracks only.

Moreover, the distribution of the Lode parameter ω at onset of damage in the specimen's center depending on loading conditions is shown in Fig. 11. In particular, for tensile loading with $F_1 : F_2 = 0 : 1$ the Lode parameter is nearly constant in the vertical section with $\omega = -1$ which is characteristic for uniaxial tension. In addition, for shear loading with $F_1 : F_2 = 1 : 0$ also nearly constant Lode parameter in the vertical section is numerically predicted with $\omega = 0$ and small zones with negative values can be seen at the boundaries. However, combined shear-tension loading will lead to more inhomogeneous distribution of the Lode parameter in the vertical section of the specimen at the onset of damage. For example, for $F_1 : F_2 = 1 : 1$ different Lode parameters between $\omega = 0.0$ and $\omega = -0.5$ are numerically predicted. The effect of superimposed tension force is much smaller for $F_1 : F_2 = 1 : 0.5$ and the distribution of the Lode parameter at onset of damage becomes again more homogeneous with maximum $\omega = -0.1$ in the specimens center. In addition, also small inhomogeneity in distribution of the Lode parameter is numerically predicted for shear-compression loading ($F_1 : F_2 = 1 : -0.5$) with maximum $\omega = 0.2$.

The stress triaxialities η covered by experiments with different flat specimens manufactured from thin sheets are shown in Fig. 12. In particular, for unnotched dog-bone-shape specimens (green) nearly homogeneous stress states occur in the small part during uniaxial tension tests with stress triaxiality $\eta = 1/3$. Higher stress

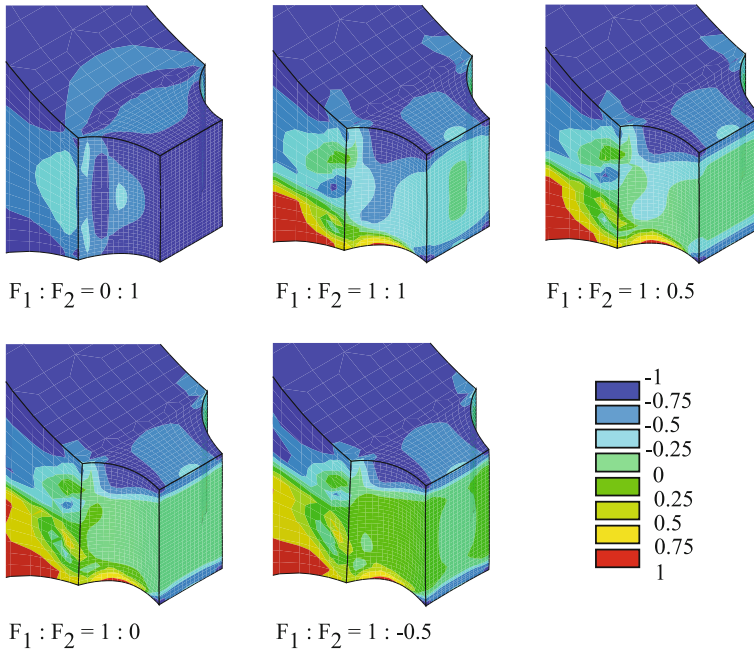
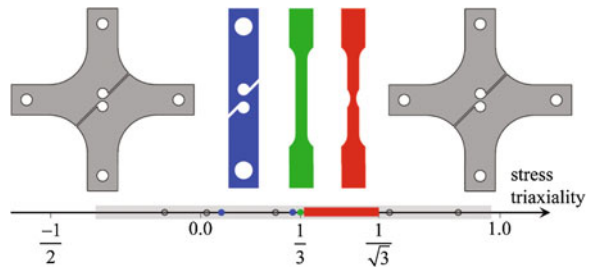


Fig. 11 Distribution of the Lode parameter ω for different load ratios

triaxialities can be obtained in uniaxial tension tests when notches with different radii are added in the middle part of the specimens (red). Decrease in notch radius will lead to an increase in stress triaxiality in the specimen's center up to $\eta = 1/\sqrt{3}$. In addition, shear specimens (blue) elongated in uniaxial tension test will lead to stress triaxialities of about $\eta = 0.1$ when notches in thickness direction are added in the central part (Brünig et al. 2008, 2011b; Driemeier et al. 2010) whereas without additional notch they will also lead to onset of damage at nearly $\eta = 1/3$. However, with these flat specimens taken from thin sheets elongated in uniaxial tension tests only the stress triaxialities shown in Fig. 12 (green, red and blue points) can be taken into account whereas no information is obtained for high positive ($\eta > 1/\sqrt{3}$), low positive (between $0.1 < \eta < 1/3$) or negative stress triaxialities. However, further experiments with new specimens (grey) tested under biaxial loading conditions discussed in the present paper will lead to stress triaxialities in the requested regimes. The grey points shown in Fig. 12 correspond to the loading conditions discussed above but variation of the load ratios $F_1 : F_2$ may lead to stress triaxialities marked by the grey zone shown in Fig. 12. Therefore, biaxial tests with 2D specimens presented in this paper cover a wide range of stress triaxialities and Lode parameters. Comparison of the experimental results and the corresponding numerical data are used to validate the stress-state-dependent damage criterion (4) and the corresponding parameters (11)–(14) for the aluminum alloy under investigation.

Fig. 12 Stress triaxialities covered by different specimens



6 Conclusions

A continuum model taking into account stress-state-dependent damage criteria and damage evolution laws has been discussed. Parameters depending on stress triaxiality and Lode parameter have been proposed based on numerical results from unit-cell calculations on the micro-level. Since the functions are only based on numerical analyses validation of the stress-state-dependent model was required. In this context, a series of new experiments with biaxially loaded specimens has been proposed. Different load ratios led to shear-tension and shear-compression mechanisms with different fracture modes. Corresponding finite element simulations of the experiments revealed a wide range of stress triaxialities and Lode parameters covered by the tests depending on biaxial loading conditions and allowed validation of the proposed stress-state-dependent functions of the continuum model.

References

- Bai Y, Wierzbicki T (2008) A new model of metal plasticity and fracture with pressure and lode dependence. *Int J Plast* 24:1071–1096
- Bao Y, Wierzbicki T (2004) On the fracture locus in the equivalent strain and stress triaxiality space. *Int J Mech Sci* 46:81–98
- Becker R, Needleman A, Richmond O, Tvergaard V (1988) Void growth and failure in notched bars. *J Mech Phys Solids* 36:317–351
- Bonora N, Gentile D, Pirondi A, Newaz G (2005) Ductile damage evolution under triaxial state of stress: theory and experiments. *Int J Plast* 21:981–1007
- Brocks W, Sun DZ, Hönl A (1995) Verification of the transferability of micromechanical parameters by cell model calculations with visco-plastic material. *Int J Plast* 11:971–989
- Brünig M (2003a) An anisotropic ductile damage model based on irreversible thermodynamics. *Int J Plast* 19:1679–1713
- Brünig M (2003b) Numerical analysis of anisotropic ductile continuum damage. *Comput Methods Appl Mech Eng* 192:2949–2976
- Brünig M, Chyra O, Albrecht D, Driemeier L, Alves M (2008) A ductile damage criterion at various stress triaxialities. *Int J Plast* 24:1731–1755
- Brünig M, Albrecht D, Gerke S (2011a) Modelling of ductile damage and fracture behavior based on different micro-mechanisms. *Int J Damage Mech* 20:558–577

- Brünig M, Albrecht D, Gerke S (2011b) Numerical analyses of stress-triaxiality-dependent inelastic deformation behavior of aluminium alloys. *Int J Damage Mech* 20:299–317
- Brünig M, Gerke S (2011) Simulation of damage evolution in ductile metals undergoing dynamic loading conditions. *Int J Plast* 27:1617–1998
- Brünig M, Gerke S, Hagenbrock V (2013) Micro-mechanical studies on the effect of the stress triaxiality and the lode parameter on ductile damage. *Int J Plast* 50:49–65
- Brünig M, Gerke S, Hagenbrock V (2014) Stress-state-dependence of damage strain rate tensors caused by growth and coalescence of micro-defects. *Int J Plast* 63:49–63
- Chew H, Guo T, Cheng L (2006) Effects of pressure-sensitivity and plastic dilatancy on void growth and interaction. *Int J Solids Struct* 43:6380–6397
- Driemeier L, Brünig M, Micheli G, Alves M (2010) Experiments on stress-triaxiality dependence of material behavior of aluminum alloys. *Mech Mater* 42:207–217
- Dunand M, Mohr D (2011) On the predictive capabilities of the shear modified Gurson and the modified Mohr-Coulomb fracture models over a wide range of stress triaxialities and lode angles. *J Mech Phys Solids* 59:1374–1394
- Gao X, Zhang G, Roe C (2010) A study on the effect of the stress state on ductile fracture. *Int J Damage Mech* 19:75–94
- Gurson AL (1977) Continuum theory of ductile rupture by void nucleation and growth: part I—yield criteria and flow rules for porous ductile media. *J Eng Mater Technol* 99:2–15
- Kuna M, Sun D (1996) Three-dimensional cell model analyses of void growth in ductile materials. *Int J Fract* 81:235–258
- Lemaitre J (1996) *A course on damage mechanics*. Springer, Berlin
- Mohr D, Henn S (2007) Calibration of stress-triaxiality dependent crack formation criteria: a new hybrid experimental-numerical method. *Exp Mech* 47:805–820
- Needleman A, Kushner A (1990) An analysis of void distribution effects on plastic flow in porous solids. *Eur J Mech A Solids* 9:193–206
- Voyiadjis G, Kattan P (1999) *Advances in damage mechanics: metals and metal matrix composites*. Elsevier, Amsterdam
- Zhang K, Bai J, Francois D (2001) Numerical analysis of the influence of the lode parameter on void growth. *Int J Solids Struct* 38:5847–5856

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